

Generalized Parton Distributions and wide-angle exclusive scattering

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The handbag mechanism for wide-angle exclusive scattering reactions is discussed and compared with other theoretical approaches. Its application to Compton scattering, meson photoproduction and two-photon annihilations into pairs of hadrons is reviewed.

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Recently a new approach to wide-angle Compton scattering off protons has been proposed [1,2] where, for Mandelstam variables $s, -t, -u$ that are large as compared to a typical hadronic scale, Λ^2 of the order of 1 GeV^2 , the process amplitudes factorize into a hard parton-level subprocess, Compton scattering off quarks, and in soft form factors which represent $1/x$ moments of generalized parton distributions (GPDs) and encode the soft physics. Subsequently it has been realized that this so-called handbag mechanism applies to a number of other wide-angle reactions such as two-photon annihilations into pairs of hadrons or meson photo- and electroproduction.

There are competing mechanisms which contribute to wide-angle scattering besides the handbag which is characterized by one active parton, i.e. one parton from each hadron participates in the hard subprocess (e.g. $\gamma q \rightarrow \gamma q$ in Compton scattering) while all others are spectators. First there are the so-called cat's ears graphs (see Fig. 1) with two active partons. It can be shown that in these graphs either a large parton virtuality or a large parton transverse momentum occurs which forces the exchange of at least one hard gluon. Hence, the cat's ears contribution is expected to be suppressed as compared to the handbag one. The next class of graphs are characterized by three active quarks and, obviously, require the exchange of at least two hard gluons. In the valence quark approximation which is expected to hold at large $-t$, the big blob in the case of three active quarks decays into two smaller blobs, see

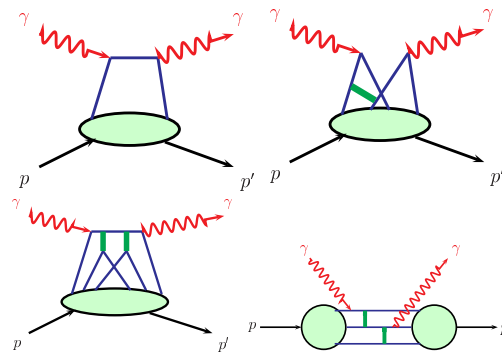


Figure 1. Handbag diagram for Compton scattering (upper left), cat's ears (upper right), the three-particle contribution (lower left) and its valence quark approximation (lower left).

Fig. 1. Each of these blobs describe a hadron's distribution amplitude for finding valence quarks in the hadron, each carrying some fraction x_i of the hadron's momentum. This so-called leading-twist contribution is expected to dominate for asymptotically large momentum transfer but is strongly suppressed for momentum transfer of the order of 10 GeV^2 . Since hadrons are not just made off their valence quarks one go on and consider four or more active partons. In principle, all the different contributions have to be added coherently. In practice, however, this is a difficult, currently impossible task since each contribution

has its own associated soft hadronic matrix element which, as yet, cannot be calculated from QCD.

The contribution from the handbag diagram shown in Fig. 1, is calculated in a symmetrical frame which is a c.m.s. rotated in such a way that the momenta of the incoming (p) and outgoing (p') proton momenta have the same light-cone plus components. Hence, the skewness defined as

$$\xi = \frac{(p - p')^+}{(p + p')^+}, \quad (1)$$

is zero. The crucial assumption in the handbag approach is that of restricted parton virtualities, $k_i^2 < \Lambda^2$, and of intrinsic transverse parton momenta, $\mathbf{k}_{\perp i}$, defined with respect to their parent hadron's momentum, which satisfy $k_{\perp i}^2/x_i < \Lambda^2$, where x_i is the momentum fraction parton i carries.

One can then show [2] that the subprocess Mandelstam variables \hat{s} and \hat{u} are the same as the ones for the full process (s and u) up to corrections of order Λ^2/t . The active partons are approximately on-shell, move collinear with their parent hadrons and carry a momentum fraction close to unity, $x_j, x'_j \simeq 1$. Thus, the physical situation is that of a hard parton-level subprocess and a soft emission and reabsorption of quarks from the proton. The arguments for handbag factorization hold in the time-like region as well [3] (see also Ref. [4]). Here a suitable symmetrical frame is a c.m.s. in which the final state hadrons move in opposite directions along the 1-axis. Hence, $p^+ = p'^+$ and the time-like skewness, defined as

$$\zeta = \frac{p^+}{(p + p')^+}, \quad (2)$$

is 1/2.

The light-cone helicity amplitudes for wide-angle Compton scattering read [2,5]

$$\begin{aligned} M_{\mu'\nu',\mu\nu}(s,t) = & \frac{e^2}{2} \left[\delta_{\nu'\nu} T_{\mu'\nu,\mu\nu} (R_V + R_A) \right. \\ & + \delta_{\nu'\nu} T_{\mu'-\nu,\mu-\nu} (R_V - R_A) \\ & + \delta_{-\nu'\nu} \frac{\sqrt{-t}}{2m} (T_{\mu'-\nu',\mu\nu} + T_{\mu'\nu',\mu-\nu}) R_T \left. \right]. \end{aligned} \quad (3)$$

The amplitudes for other wide-angle reaction have an analogue structure. $\mu(\nu)$ and $\mu'(\nu')$ de-

note the helicities of the incoming and outgoing photons (protons in M or quarks in the subprocess amplitude $T(\hat{s},t)$), respectively. m denotes the mass of the proton. The form factors $R_i(t)$ represent $1/\bar{x}$ -moments of GPDs at zero skewness. For Compton scattering the hard scattering has been calculated to next-to-leading order perturbative QCD [5]. To this order one has to take into account the photon-gluon subprocess and a corresponding gluonic form factor. This small correction which amounts to less than 10% in the cross section, is taken into account in the numerical results shown below but, for convenience, ignored in the formulas.

The handbag amplitude (3) leads to the following leading-order Compton cross section

$$\begin{aligned} \frac{d\sigma}{dt} = & \frac{d\hat{\sigma}}{dt} \left\{ \frac{1}{2} [R_V^2(t) (1 + \kappa_T^2) + R_A^2(t)] \right. \\ & \left. - \frac{\hat{u}\hat{s}}{\hat{s}^2 + \hat{u}^2} [R_V^2(t) (1 + \kappa_T^2) - R_A^2(t)] \right\}, \end{aligned} \quad (4)$$

where $d\hat{\sigma}/dt$ is the Klein-Nishina cross section for Compton scattering off massless, point-like spin-1/2 particles of charge unity. The quantity κ_T is defined as

$$\kappa_T = \frac{\sqrt{-t}}{2m} \frac{R_T}{R_V}. \quad (5)$$

Another interesting observable in Compton scattering is the helicity correlation, A_{LL} , between the initial state photon and proton or, equivalently, the helicity transfer, K_{LL} , from the incoming photon to the outgoing proton. In the handbag approach one obtains [2,5]

$$A_{LL} = K_{LL} \simeq \frac{\hat{s}^2 - \hat{u}^2}{\hat{s}^2 + \hat{u}^2} \frac{R_A(t)}{R_V(t)}, \quad (6)$$

where the factor in front of the form factors is the corresponding observable for $\gamma q \rightarrow \gamma q$. The result (6) is a robust prediction of the handbag mechanism, the magnitude of the subprocess helicity correlation is only diluted by the ratio of the form factors R_A and R_V .

The cross section for two-photon annihilations into baryon-antibaryon pairs reads [3]

$$\begin{aligned} \frac{d\sigma}{dt} (\gamma\gamma \rightarrow B\bar{B}) = & \frac{4\pi\alpha_{\text{elm}}^2}{s^2 \sin^2 \theta} \left\{ |R_A^B(s) + R_P^B(s)|^2 \right. \\ & \left. + \cos^2 \theta |R_V^B(s)|^2 + \frac{s}{4m^2} |R_P^B(s)|^2 \right\}, \end{aligned} \quad (7)$$

while for pseudoscalar meson it is given by

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow M\bar{M}) = \frac{8\pi\alpha_{\text{elm}}^2}{s^2 \sin^4\theta} |R_{M\bar{M}}(s)|^2. \quad (8)$$

The annihilation form factors represent moments of two-hadron distribution amplitudes, $\Phi_{2h}(z, \zeta, s)$, which are time-like versions of GPDs.

In order to make actual predictions for Compton scattering a model for the form factors or rather for the underlying GPDs is required. A first attempt to parameterize the GPDs H and \tilde{H} at zero skewness is [1,2]

$$\begin{aligned} H^q(\bar{x}, 0; t) &= \exp\left[a^2 t \frac{1-\bar{x}}{2\bar{x}}\right] q(\bar{x}), \\ \tilde{H}^q(\bar{x}, 0; t) &= \exp\left[a^2 t \frac{1-\bar{x}}{2\bar{x}}\right] \Delta q(\bar{x}), \end{aligned} \quad (9)$$

where $q(\bar{x})$ and $\Delta q(\bar{x})$ are the usual unpolarized and polarized parton distributions in the proton. The transverse size of the proton, a , is the only free parameter. The model (9) is designed for large $-t$. Hence, forced by the exponential in (9), large \bar{x} is implied, too. Despite this the normalizations of the model GPDs at $t = 0$ are correct.

From the model GPDs (9) we can evaluate the various form factors by taking appropriate moments. For the Dirac and the axial form factor one has the sum rules

$$\begin{aligned} F_1(t) &= \sum_q e_q \int_{-1}^1 d\bar{x} H^q(\bar{x}, 0; t), \\ F_A(t) &= \int_{-1}^1 d\bar{x} [\tilde{H}^u(\bar{x}, 0; t) - \tilde{H}^d(\bar{x}, 0; t)] \end{aligned} \quad (10)$$

while the Compton form factors read

$$\begin{aligned} R_V(t) &= \sum_q e_q^2 \int_{-1}^1 \frac{d\bar{x}}{\bar{x}} H^q(\bar{x}, 0; t), \\ R_A(t) &= \sum_q e_q^2 \int_{-1}^1 \frac{d\bar{x}}{\bar{x}} \text{sign}(\bar{x}) \tilde{H}^q(\bar{x}, 0; t). \end{aligned} \quad (11)$$

Numerical results for the proton's Dirac form factor and the Compton form factors are shown in Fig. 2. The scaled form factors $t^2 F_{1,A}$ and $t^2 R_i$ exhibit broad maxima which mimick dimensional counting in a range of $-t$ from, say, 5 to about 20 GeV^2 . As the comparison with experiment [6]

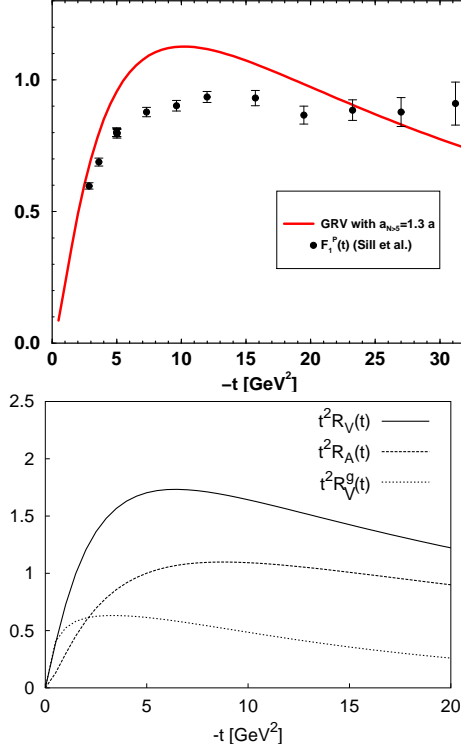


Figure 2. Model results for the scaled Dirac form factor of the proton $t^2 F_1^p$ (top) and the Compton form factors $t^2 R_i$ (bottom) (in GeV^4). Data taken from Ref. [6].

reveals, the model GPDs work rather well although the predictions for the Dirac form factor overshoot the data by about 20 – 30 % for $-t$ around 5 GeV^2 . An effect of similar size can be expected for the Compton form factors. A phenomenological analysis of the GPDs is currently in progress. It bases on a more general ansatz for the general profile function in (9) with a few free parameters which are adjusted to the data on the nucleon form factors through the sum rules. In this analysis the GPD E^q being related to nucleon helicity flip, will also be determined. With it at our disposal one can evaluate the tensor form factor R_T analogously to (11). In the present stage only predictions for various constant values of the quantity κ_T (5) are given.

It is important to realize that the GPDs represent process independent information on the nu-

cleon. They therefore appear for instance in meson photoproduction as well. However, the flavor composition of the full form factors as well as the relative sign between quark and antiquark contributions are process dependent. In time-like processes form factors occur which are analytic continuations of the above space-like ones. Thus, for instance, in $\gamma\gamma \rightarrow p\bar{p}$ we have

$$R_i(s) = \sum_q e_q^2 F_i^q(s), \quad i = V, A, P \quad (12)$$

with

$$F_i^q(s) = \int dz \Phi_{p\bar{p}i}^q(z, \zeta = 1/2, s). \quad (13)$$

In contrast to the space-like region where the pseudoscalar form factor decouples in the symmetric frame, here the scalar form factor does not contribute. The vector form factor is therefore related to the magnetic one

$$G_M(s) = \sum_q e_q F_V^q(s), \quad (14)$$

and not to the Dirac form factor. It is expected that the time-like form factors are of the same order of magnitude than the space-like ones.

Employing the model GPDs and the corresponding form factors, various Compton observables can be calculated [2,5]. The predictions for the differential cross section are in fair agreement with the Cornell data [7]. The JLab E99-114 collaboration [8] will provide accurate cross section data soon which will allow for a crucial examination of the handbag mechanism.

Predictions for $A_{LL} = K_{LL}$ are shown in Fig. 3. The JLab E99-114 collaboration [8] has presented a first measurement of K_{LL} at a c.m.s. scattering angle of 120° and a rather low photon energy of 3.23 GeV. This still preliminary data point is in fair agreement with the predictions from the handbag given the small energy at which they are available.

For photo- and electroproduction of mesons the dynamics of the subprocess, $\gamma q \rightarrow Mq$, is to be specified. The simplest mechanism is the one-gluon exchange. It turns out that this contribution fails with the normalization of the photoproduction cross section [9]. Treating the subprocess

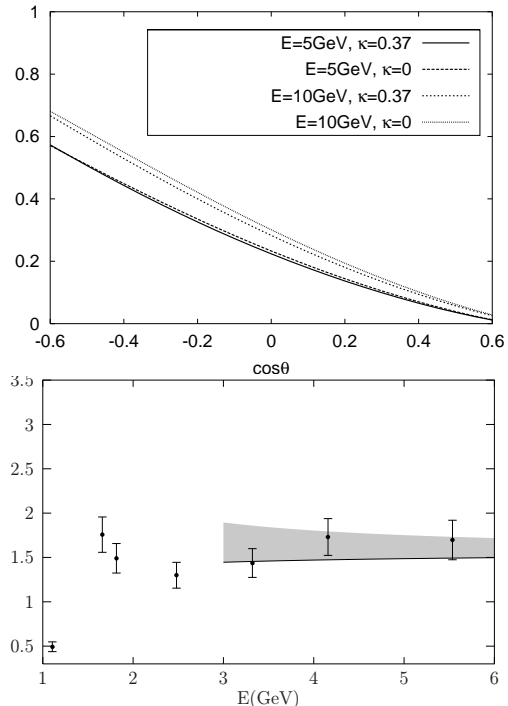


Figure 3. Predictions for the helicity correlation $A_{LL} = K_{LL}$ in Compton scattering (top) and the ratio of the $\gamma n \rightarrow \pi^- p$ and $\gamma p \rightarrow \pi^+ n$ cross sections vs. beam energy, E , at a c.m.s. scattering angle of 90° (bottom). Data taken from [10].

in a more general way by a covariant decomposition and neglecting quark helicity flip, one obtains a number of predictions whose experimental verification would signal the dominance of the handbag mechanism. Thus, for instance, the helicity correlation A_{LL} is similar to the result (6) for Compton scattering. Another interesting result is the ratio of the cross sections for photoproduction of π^+ and π^- . It is approximately given by

$$\frac{d\sigma(\gamma n \rightarrow \pi^- p)}{d\sigma(\gamma p \rightarrow \pi^+ n)} \simeq \left[\frac{e_d \hat{u} + e_u \hat{s}}{e_u \hat{u} + e_d \hat{s}} \right]^2. \quad (15)$$

The form factors cancel in the ratio. The prediction (15) is in fair agreement with a recent JLab measurement [10], see Fig. 3.

The characteristic $\sin^{-4}\theta$ dependence of the cross section for two-photon annihilation into a pair of pseudoscalar mesons (8) is predicted in fair agreement with experiment, see Fig. 4. The

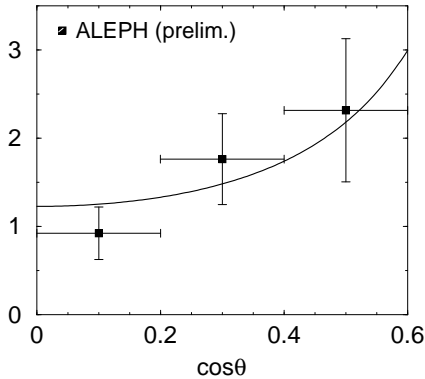


Figure 4. Handbag predictions for the angular dependence of the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$ versus s . Preliminary data are taken from ALEPH [11].

form factors have not been modelled in Refs. [3] but rather extracted ('measured') from the experimental cross section. The average value of the scaled form factor $sR_{2\pi}$ obtained that way is 0.75 GeV^2 . The closeness of this value to that of the scaled time-like electromagnetic form factor of the pion ($0.93 \pm 0.12 \text{ GeV}^2$) hints at the internal consistency of the handbag approach. Similar results are found for the production of baryon-antibaryon pairs.

Another feature of the handbag mechanism in the time-like region is the intermediate $q\bar{q}$ state implying the absence of isospin-two components in the final state. A consequence of this property is

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^0\pi^0) = \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-), \quad (16)$$

which is independent of the soft physics input and is, in so far, a robust prediction of the handbag approach. The absence of the isospin-two components combined with flavor symmetry allows one to calculate the cross sections for other $B\bar{B}$ channels using the $p\bar{p}$ form factors as the only soft physics input.

To summarize, I have briefly reviewed the theoretical activities on applications of the handbag mechanism to wide-angle scattering. There are many interesting predictions, some are in fair agreement with experiment, others still awaiting

their experimental examination. It seems that the handbag mechanism plays an important role in exclusive scattering for momentum transfers of the order of 10 GeV^2 . However, before we can draw firm conclusions more experimental tests are needed. I finally emphasize that the structure of the handbag amplitude, namely its representation as a product of perturbatively calculable hard scattering amplitudes and $t(s)$ -dependent form factors is the essential result. Refuting the handbag approach necessitates experimental evidence against this factorization.

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